Odd Factor Decomposition of H-V Super Magic Graphs

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Abstract

An H-magic labeling in an H-decomposable graph G is a bijection $f: V(G) \cup E(G) \rightarrow \{1,2 \dots . p+q\}$ such that for every copy H in the decomposition $\sum_{v \epsilon V(H)} f(v) + \sum_{e \epsilon E(H)} f(e)$ is constant. The function f is named as H-V super magic labeling if $f\big(V(G)\big) = \{1,2, \dots p\}$.In this article we tend to impart that for few even regular and odd regular graphs to possess an $(2k+1)$-factor H-Vsuper magic decomposition, for $k \geq 1$.

Keywords: H-magic labeling, H-decomposable graph, H-factorable, H-V super magic labeling,$(2k+1)$-factor H-V-super magic decomposition.

AMS subject Classification Code:05C78

1. Introduction

In this paper we tend to discuss merely finite, simple and undirected graphs. Let G be a graph with vertex set $V(G)$ and the edge set $E(G)$ such that the order of $G = |V(G)| = p$ and the size of $G = |E(G)| = q$. A labeling of a graph G is a mapping from a set of vertices(edges) into a set of numbers, typically integers.

Various kinds of labeling are examined as well as delineated by numerous authors. An outstanding survey of graph labelings are often found in[2]. In 1963, Sedlàček[7] introduced the theory of magic labeling in graphs. A graph G is *magic* if the edges of G are often labelled by a set of numbers$\{1,2, \dots q\}$ so that the summation of labels of all the edges incident with any vertex is that the same.

A graph G is supposed to be H-decomposable if G has a family of subgraphs $H_1, H_2, .. H_h$ such that all the subgraphs are isomorphic to a graph H, $E(H_i) \cap E\big(H_j\big) = \phi$ for $j \neq i$ and $\bigcup_{i=1}^{h} E(H_i) = E(G)$. If each $H_i$is a spanning of G, then G is meant to be H-factorable. When H is a $m-$regular graph then G is meant to be a $m-$factorable. If G is a $m-$factorable graph,then essentially G is $r-$regular for certain integer r that is a multiple of m.

Super vertex magic labeling was initiated by Swaminathan and Jeyanthi[9]. A vertex magic labeling is bijection$f: V(G) \cup E(G) \rightarrow \{1,2 \dots p+q\}$ with the property, for every $u \epsilon V(G)$, $f(u) + \sum_{v \epsilon N(u)} f(uv) = k$ for some constant k. Such labeling is super vertex magic labeling if $f\big(E(G)\big) = \{1,2 \dots q\}$ and $f\big(V(G)\big) = \{1,2 \dots p\}$ A graph G is named E-super vertex magic if it possess an E-super vertex magic labeling. E-super vertex labeling of graphs has been studied by Marimuthu and Balakrishnan[4]. They verified that if a graph G of odd order can be decomposed into two Hamiltonian cycles then G is an E-super vertex magic graph. T.M Wang and Guang-Hui[10] gave the generalization of some results by means of 2-factor.

In 2010, A.A.G Ngurah,A.N.M Salman,L.Susilowati[5] established the idea H-super magic labeling of graphs. A graph G is meant to be H-magic if there exists a bijection $f: V(G) \cup E(G) \to \{1,2 \ldots |V(G) \cup E(G)|\}$ such that for every subgraph H' of G is isomorphic to H , $\sum_{v \epsilon V(H')} f(v) + \sum_{e \epsilon E(H')} f(e)$ is a constant. G is claimed to be H-super magic if $f\big(V(G)\big) = \{1,2, \ldots |V(G)|\}$.

In 2014, Subbiah and Pandimadevi[8]introduced the theory of H-E super magic. An H –magic labeling in an H-decomposable graph G is a bijection $f: V(G) \cup E(G) \to \{1,2 \ldots p+q\}$ such that for every copy H in the decomposition, $\sum_{v \epsilon V(H)} f(v) + \sum_{e \epsilon E(H)} f(e)$ is a constant.The function f is named to be H-E super magic if $f\big(E(G)\big) = \{1,2 \ldots q\}$.They gave some basic properties of H-E super magic labeling and provided the essential and adequate conditions for 2-factor decomposition of H-E magic graphs.

In 2016, P.Hemalatha and V.Nivetha[3] discussed the idea of Odd Decomposition of E-super magic graphs. They gave an essential and adequate condition for few even regular and odd regular graphs G to possess an $(2k+1)$- factor E super magic decomposition for $k \geq 1$.

In this article, we tend to prove for a few regular graphs to possess $(2k+1)$- factor H-V super magic decomposition. The following solutions are helpful in proving the theorem.

Lemma 1.1[4]

If a non-trivial graph G with order p and size q is super vertex magic, then the magic constant k is given by $k = q + \frac{p+1}{2} + \frac{q(q+1)}{2s}$

Lemma1.2[8]

If a non-trivial m-factor decomposable graph G is m-factor E-super magic decomposable,then the sum of the edge labels, denoted by k_e is a constant and it is given by $k_e = \frac{q(q+1)}{2s}$ where s is the number of m- factors of G

Lemma 1.3[6]

Every m and r-regular graph has r-factor for every positive integer r and m ,where $r, m \geq 1$

2.Odd Factor Decomposition of H-V-Super Magic Graphs

In this section an essential and adequate condition is obtained for few even or odd regular graphs G of even order to possess an $(2k+1)$-factor H-V-super magic decomposition for $k \geq 1$.

Theorem 2.1

Every $(8k^2-2)$ regular graph G of order $8k^2$ is $(2k+1)$ -factor H-V-super magic decomposable for $k \geq 1$

Proof:

Since G is a $(8k^2-2)$ regular graph of order $p = 8k^2$,we have $q = (8k^2-2)(4k^2)$. Also the number of $(2k+1)$ factor is $h = 4k-2$.

Let $F_1, F_2, \dots F_h$ be the factors of G.

Table1

F_1	F_2	....	F_h
$1+p$	$2+p$	....	$h+p$
$2h+p$	$2h+p-1$	....	$h+p+1$
$2h+p+1$	$2h+p+2$	....	$3h+p$
$4h+p$	$4h+p-1$	....	$3h+p+1$
....	....	....	....
$\left[\frac{(2k+1)p-4}{2}\right]h+p+1$	$\left[\frac{(2k+1)p-4}{2}\right]h+p+2$	....	$\left[\frac{(2k+1)p-2}{2}\right]h+p$
$\left[\frac{(2k+1)p}{2}\right]h+p$	$\left[\frac{(2k+1)p}{2}\right]h+p-1$	....	$\left[\frac{(2k+1)p-2}{2}\right]h+p+1$

From the above table, the sum of the edge labels of each factor F_i, when i is odd, is calculated as follows:

$$K_e = \sum f(F_i) \qquad = p+i+2h+p-(i-1)+2h+p+i+4h+p+\cdots\left[\frac{(2k+1)p}{2}\right]h+p+i+\left[\frac{(2k+1)p}{2}\right]h+p-(i-1)$$

$$= 2[2h+4h+\cdots+(p-2)h]+\left[1+1+\cdots+\frac{(2k+1)p}{4}\right]+\frac{(2k+1)ph}{2}+\left[\frac{(2k+1)p}{2}\right]p$$

$$= 4h\left[1+2+\cdots.+\frac{(2k+1)p-4}{4}\right]+\frac{(2k+1)p}{4}+\frac{(2k+1)ph}{2}+\left[\frac{(2k+1)p}{2}\right]p$$

$$= \frac{h}{8}[(2k+1)p]^2+\frac{(2k+1)p}{4}+\left[\frac{(2k+1)p}{2}\right]p$$

$$= \frac{(2k+1)p}{8}[(2k+1)ph+2+4p]$$

$$= \frac{(2k+1)(8k^2)}{8}[(2k+1)(8k^2)(4k-2)+2+4(8k^2)]$$

$$K_e = 128k^7+64k^6+32k^5+16k^4+4k^3+2k^2$$

Likewise, we can compute $\sum f(F_i)$ for each factor F_i when i is even.

Hence G is $(2k+1)$- factor H-V-super magic decomposable.

Example 2.2

A 6-regular graph G of order 8 is 3-factor-H-V super magic decomposable.

Here k=1,h=2

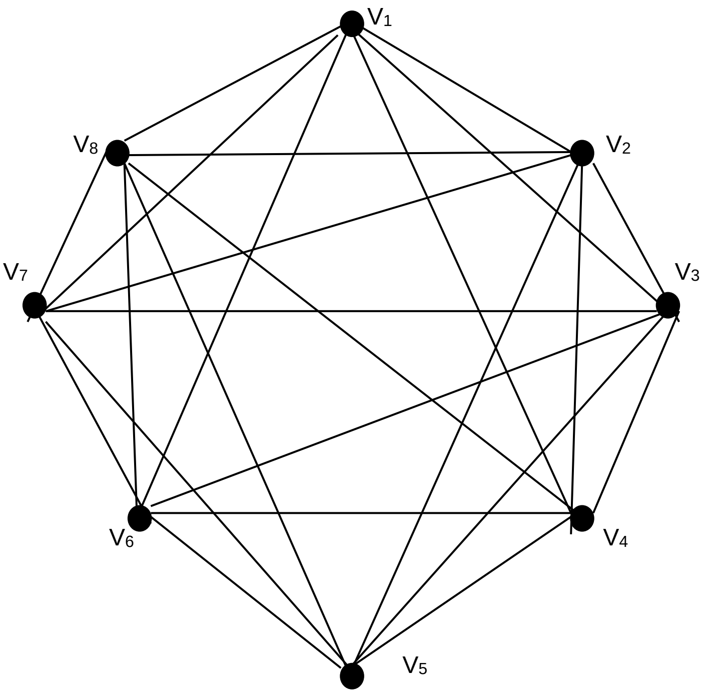

Fig 1 G

The graphs G is decomposed into two 3-factor i.e,F_1 and F_2

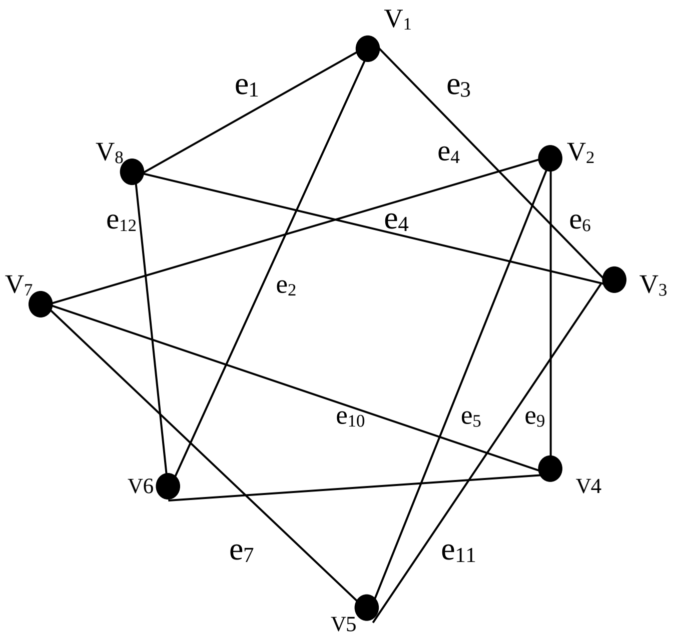

Fig 2 F_1

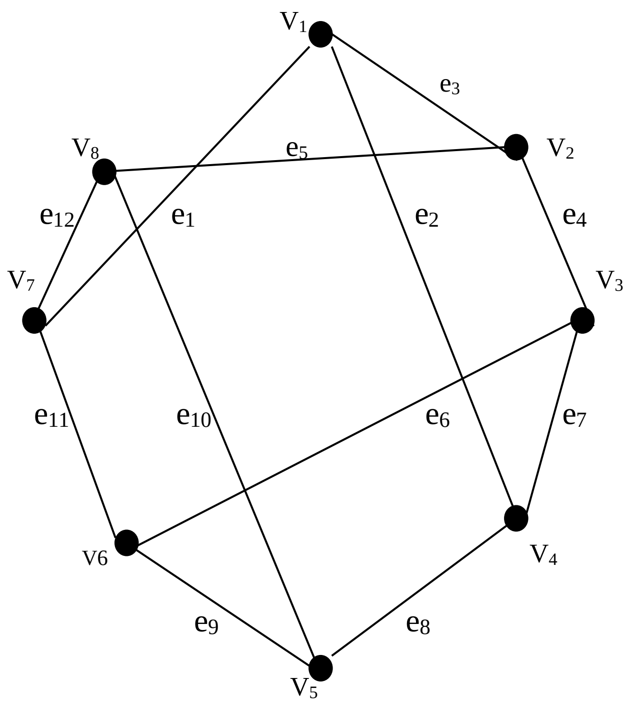

Fig 3 F_2

Table 2

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	K_e
F_1	9	12	13	16	17	20	21	24	25	28	29	32	246
F_2	10	11	14	15	18	19	22	23	26	27	30	31	246

Here $\sum f(F_i) = K_e = 246$ for i=1,2

Magic Squares

Magic squares are among the more prevalent mathematical recreations. A classical reference on magic squares is [1].A magic square of side n is an $n \times n$ array whose entries are an arrangement of integers {1,2,...n^2}in which all components in any row, any column or either main diagonal or back-diagonal, raise to equivalent sum. Furthermore, we denote this sum as magic number(MN)and also we observe that the value of the magic number is $MN = \frac{1}{2}n(n^2 + 1)$.

Theorem 2.3

Every $(12k^2 - 3)$ regular graph G of order $(12k^2 - 2)$ is $(2k + 1)$- factor H-V-super magic decomposable for $k \geq 1$

Proof:

Since G is a $(12k^2 - 3)$ regular graph of order $p = (12k^2 - 2)$,we have
$q = (12k^2 - 3)(6k^2 - 1)$. Also the number of $(2k + 1)$ factor is $h = 6k - 3$.
Let $F_1, F_2, \dots F_h$ be the factors of G.

Table 3

F_1	F_2	....	F_h
$(h \times h\ magic\ square) + p$			
h^2+p+1	h^2+p+2	....	h^2+h+p
h^2+2h+p	$h^2+2h+p-1$	....	$h^2+h+p+1$
$h^2+2h+p+1$	$h^2+2h+p+2$	....	h^2+3h+p
h^2+4h+p	$h^2+4h+p-1$	....	$h^2+3h+p+1$
....	....	....	....
$\left[\frac{(2k+1)p-4}{2}\right]h+p+1$	$\left[\frac{(2k+1)p-4}{2}\right]h+p+2$	....	$\left[\frac{(2k+1)p-2}{2}\right]h+p$
$\left[\frac{(2k+1)p}{2}\right]h+p$	$\left[\frac{(2k+1)p}{2}\right]h+p-1$	....	$\left[\frac{(2k+1)p-2}{2}\right]h+p+1$

From the above table, the sum of the edge labels of each factor F_i, when i is odd, is calculated as follows:

$$K_e = \sum f(F_i) = MN + h^2 + p + i + h^2 + 2h + p - (i-1) + h^2 + 2h + p + 1 + h^2 + 4h + p - (i-1) + \ldots\ldots + \left[\frac{(2k+1)p}{2}\right] h + p + i + \left[\frac{(2k+1)p}{2}\right] h + p - (i-1)$$

where MN is magic number of $h \times h$ magic squares

$$= \frac{1}{2}h(h^2+1) + ph + \left[\frac{(2k+1)p-2h}{2}\right] h^2 + 2\left(2h + 4h + \cdots + \left[\frac{(2k+1)p-2h-4}{2}\right] h\right) + (1+1+\cdots+1) + \left[\frac{(2k+1)p-2h}{2}\right] h + \left[\left[\frac{(2k+1)p}{2}\right] p\right]$$

$$= \frac{h^3}{2} + \frac{h}{2} + \frac{(2k+1)ph^2}{2} - h^3 + \frac{((2k+1)p)^2 h}{8} - \frac{(2k+1)ph^2}{2} + \frac{h^3}{2} - \frac{(2k+1)ph}{2} + h^2 + \frac{(2k+1)p}{4} - \frac{h}{2} + \frac{(2k+1)ph}{2} - h^2 + \left[\frac{(2k+1)p}{2}\right] p$$

$$= \frac{(2k+1)p}{8}[(2k+1)ph + 2 + 4p]$$

$$= \frac{(2k+1)(12k^2-2)}{8}[(2k+1)(12k^2-2)(6k-3) + 2 + 4(12k^2-2)]$$

$$K_e = 432k^7 + 216k^6 - 108k^5 - 54k^4 + 6k^3 + 3k^2$$

Likewise, we can compute $\sum f(F_i)$ for each factor F_i when i is even.

Hence G is $(2k+1)$- factor H-V-super magic decomposable.

Example 2.4

A 9-regular graph G of order 10 is 3-factor V-super magic decomposable

Here k=1,p=10 and also h=3

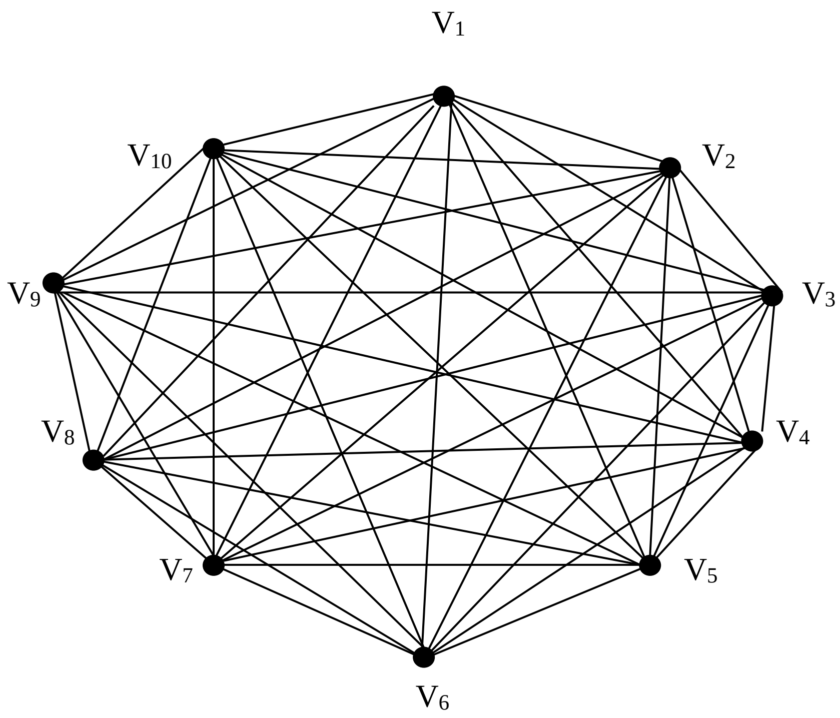

Fig 4 G

The graphs G is decomposed into three 3-factor i.e, F_1 , F_2 and F_3

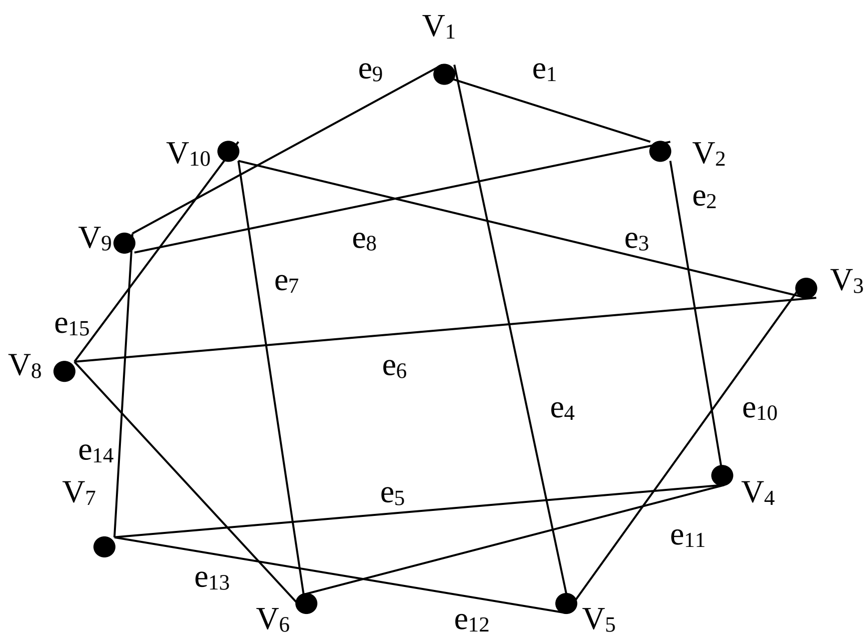

Fig 5 F_1

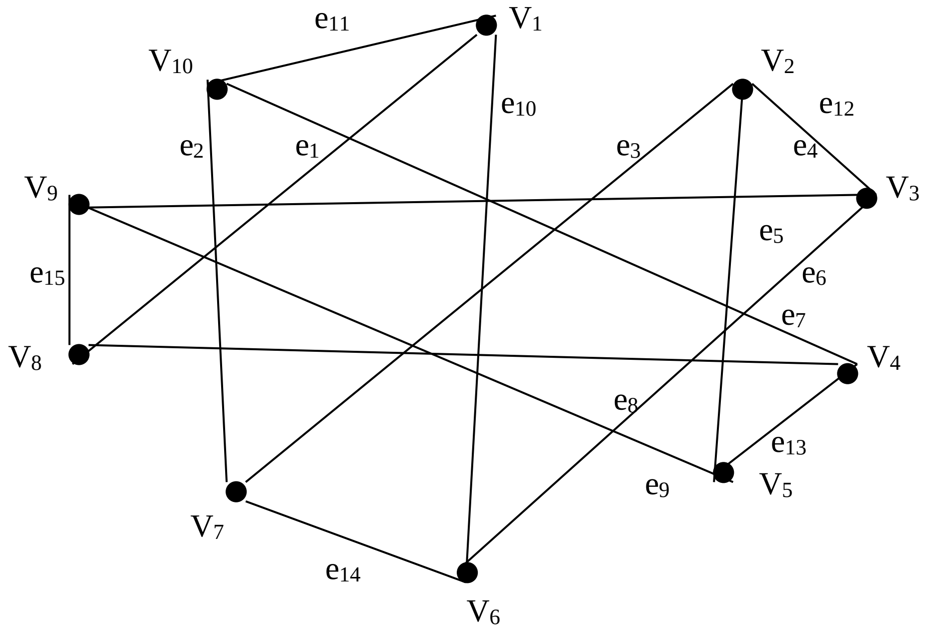

Fig 6 F_2

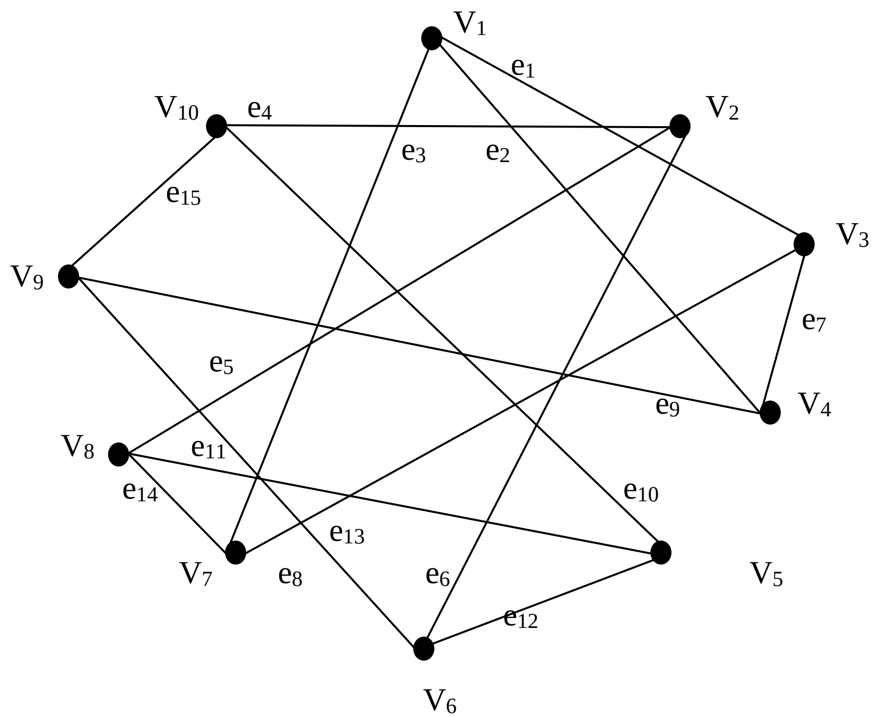

Fig 7 F_3

Table 4

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
F_1	14	13	18	20	25	26	31	32	37	38
F_2	19	15	11	21	24	27	30	33	36	39
F_3	12	17	16	22	23	28	29	34	35	40

	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	K_e
F_1	43	44	49	50	55	495
F_2	42	45	48	51	54	495
F_3	41	46	47	52	53	495

From the table,the sum of the edge labels of each factor is $K_e = 495$

3.Conclusion

In this article, we tend to obtained for few even regular and odd regular graphs of even order to possess an (2k+1)-factor H-V-super magic decomposition for $k \geq 1$. In future H-V anti-magic decomposition may be performed.

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