ON GENERALIZED FERMAT EQUATIONS INVOLVING JARASANDHA NUMBERS

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Abstract:

In this paper, we search for integer solutions to the generalized Fermat equations involving jarasandha numbers. A few interesting relations between the solutions are also presented.

Keywords:

Jarasandha numbers, Diophantine equation, integer solutions, Generalized Fermat equations.

Introduction:

Number theory is captivating in light of the fact that it has such an enormous number of open problems that seem accessible from the outside. Obviously, open issues in number theory are open which is as it should be. Numbers, regardless of being basic, have an inconceivably rich structure which we just comprehend to a certain point. In the mid twentieth century, Thue made a significant achievement in the investigation of diophantine equations. His proof is one of the primary instances of the polynomial strategy. His proof impacted a great deal of later work in number theory, including diophantine equations. So number theory and its distinctive subfields will continue exciting the minds of mathematicians for an extremely prolonged stretch of time [1-3].

In our Indian epic Mahabharatha, we come across a Person named 'jarasandha'. He had a boon that if he was split into two parts and thrown apart, the parts would rejoin and come back to life. In the field of Mathematics, we have numbers exhibiting the same property as Jarasandha [4-13]. In[14], generalized Fermat equations are analysed. These outcomes propelled us to analyze for integer solutions to Fermat equations involving Jarasandha numbers. In this communication, we consider generalized Fermat equations involving jarasandha numbers and obtain infinitely many solutions.

Method of Analysis:

In this paper, we present three different set of solutions to the generalised Fermat equations,

$$x^{\sqrt{J}+1} + y^{\sqrt{J}-1} = z^{J} \tag{1}$$

where J is a Jarasandha number.

Set 1:

Assuming

$x = n^{\alpha}(n^{\alpha}+1)^{\sqrt{J}-1},\ y = n^{\beta}(n^{\alpha}+1)^{\sqrt{J}+1},\ z = n^{\gamma}(n^{\alpha}+1)$ in (1) we get

$$n^{(\sqrt{J}+1)\alpha} + n^{(\sqrt{J}-1)\beta} = n^{J\gamma+\alpha} + n^{J\gamma}$$

On equating the exponents on both sides, we have

$$(\sqrt{J}+1)\alpha = J\gamma + \alpha$$

$$(\sqrt{J}-1)\beta = J\gamma$$

The above system is satisfied when

$$\alpha = \sqrt{J}(\sqrt{J}-1)s, \quad \beta = Js, \quad \gamma = (\sqrt{J}-1)s$$

Hence the values of x, y & z satisfying (1) are given by,

$$x = n^{\sqrt{J}(\sqrt{J}-1)s}(n^{\sqrt{J}(\sqrt{J}-1)s}+1)^{\sqrt{J}-1}$$

$$y = n^{Js}(n^{\sqrt{J}(\sqrt{J}-1)s}+1)^{\sqrt{J}+1}$$

$$z = n^{(\sqrt{J}-1)s}(n^{\sqrt{J}(\sqrt{J}-1)s}+1)$$

Some numerical examples for the jarasandha numbers are exhibited in the following table:

S.No	J	n	s	x	y	z
1.	81	2	1	$2^{72}(2^{72}+1)^{8}$	$2^{81}(2^{72}+1)^{10}$	$2^{8}(2^{72}+1)$
2.	2025	2	1	$2^{1980}(2^{1980}+1)^{44}$	$2^{2025}(2^{1980}+1)^{46}$	$2^{44}(2^{1980}+1)$
3.	3025	3	1	$3^{2970}(3^{2970}+1)^{54}$	$3^{3025}(3^{2970}+1)^{56}$	$3^{54}(3^{2970}+1)$
4.	9801	2	1	$2^{9702}(2^{9702}+1)^{98}$	$2^{9801}(2^{9702}+1)^{100}$	$2^{98}(2^{9702}+1)$
5.	88209	2	1	$2^{87912}(2^{87912}+1)^{296}$	$2^{88209}(2^{87912}+1)^{298}$	$2^{296}(2^{87912}+1)$

Properties:

1. $\frac{n^{\alpha-\beta+2\gamma}y}{x}$ is a perfect square.
2. $n^{\alpha-\beta+2\gamma}y = xz^2$.
3. $\frac{6n^{\beta-\alpha}y}{x}$ is a nasty number.
4. $xz^{\alpha} = y^{\gamma}$.

Set 2:

Assuming

$x = n^{\alpha}(n^{\beta}+1)^{\sqrt{J}-1}$, $y = n^{\beta}(n^{\beta}+1)^{\sqrt{J}+1}$, $z = n^{\gamma}(n^{\beta}+1)$ in (1) we get

$$n^{(\sqrt{J}+1)\alpha} + n^{(\sqrt{J}-1)\beta} = n^{J\gamma+\beta} + n^{J\gamma}$$

On equating the exponents on both sides, we have

$$(\sqrt{J}+1)\alpha = J\gamma$$

$$(\sqrt{J}-1)\beta = J\gamma + \beta$$

The above system is satisfied when

$$\alpha = \sqrt{J}(\sqrt{J}-2)s, \quad \beta = J(\sqrt{J}+1)s, \quad \gamma = (\sqrt{J}+1)(\sqrt{J}-2)s$$

Hence the values of x, y & z satisfying (1) are given by,

$$x = n^{J(\sqrt{J}-2)s}(n^{J(\sqrt{J}+1)s}+1)^{\sqrt{J}-1}$$
$$y = n^{J(\sqrt{J}+1)s}(n^{J(\sqrt{J}+1)s}+1)^{\sqrt{J}+1}$$
$$z = n^{(\sqrt{J}+1)(\sqrt{J}-2)s}(n^{J(\sqrt{J}+1)s}+1)$$

Some numerical examples for the jarasandha numbers are exhibited in the following table:

S.No	J	n	s	x	y	z
1.	81	2	1	$2^{567}(2^{810}+1)^{8}$	$2^{810}(2^{810}+1)^{10}$	$2^{70}(2^{810}+1)$
2.	2025	3	1	$3^{87075}(3^{93150}+1)^{44}$	$3^{93150}(3^{93150}+1)^{46}$	$3^{1978}(3^{93150}+1)$
3.	3025	2	1	$2^{160325}(2^{169400}+1)^{54}$	$2^{169400}(2^{169400}+1)^{56}$	$2^{2968}(2^{169400}+1)$
4.	9801	3	1	$3^{950697}(3^{980100}+1)^{98}$	$3^{980100}(3^{980100}+1)^{100}$	$3^{9700}(3^{980100}+1)$
5.	88209	2	1	$2^{26021655}(2^{26286282}+1)^{296}$	$2^{26286282}(2^{26286282}+1)^{298}$	$2^{87910}(2^{26286282}+1)$

Properties:

1. $\frac{yz}{n^{\gamma}x}$ is a cubical integer.
2. $n^{\alpha-\beta-2\gamma}y = xz^2$.
3. $\frac{6n^{\beta-\alpha}y}{xz^2}$ is a nasty number.
4. $\frac{n^{\beta-\alpha}y}{x}$ is a perfect square.

Set 3:

Assuming

$x = n^{\alpha}(n^{\gamma}+1)^{\sqrt{J}-1}$, $y = n^{\beta}(n^{\gamma}+1)^{\sqrt{J}+1}$, $z = n^{\gamma}(n^{\gamma}+1)$ in (1) we get

$$n^{(\sqrt{J}+1)\alpha} + n^{(\sqrt{J}-1)\beta} = n^{J\gamma+\gamma} + n^{J\gamma}$$

On equating the exponents on both sides, we have

$$(\sqrt{J}+1)\alpha = (J+1)\gamma$$

$$(\sqrt{J}-1)\beta = J\gamma$$

The above system is satisfied when

$$\alpha = (J+1)(\sqrt{J}-1)s,\quad \beta = J(\sqrt{J}+1)s,\quad \gamma = (J-1)s$$

Hence the values of x, y & z satisfying (1) are given by,

$$x = n^{(J+1)(\sqrt{J}-1)s}(n^{(J-1)s}+1)^{\sqrt{J}-1}$$
$$y = n^{J(\sqrt{J}+1)s}(n^{(J-1)s}+1)^{\sqrt{J}+1}$$
$$z = n^{(J-1)s}(n^{(J-1)s}+1)$$

Some numerical examples for the jarasandha numbers are exhibited in the following table:

S.No	J	n	s	x	y	z
1.	81	2	1	$2^{656}(2^{80}+1)^{8}$	$2^{810}(2^{80}+1)^{10}$	$2^{80}(2^{80}+1)$
2.	2025	2	1	$2^{89144}(2^{2024}+1)^{44}$	$2^{93150}(2^{2024}+1)^{46}$	$2^{2024}(2^{2024}+1)$
3.	3025	3	1	$3^{163404}(3^{3024}+1)^{54}$	$3^{169400}(3^{3024}+1)^{56}$	$3^{3024}(3^{3024}+1)$
4.	9801	3	1	$3^{960596}(3^{9800}+1)^{98}$	$3^{980100}(3^{9800}+1)^{100}$	$3^{9800}(3^{9800}+1)$
5.	88209	3	1	$3^{26110160}(3^{88208}+1)^{296}$	$3^{26286282}(3^{88208}+1)^{298}$	$3^{88208}(3^{88208}+1)$

Properties:

1. $\dfrac{n^{\alpha-\beta+2\gamma}y}{x}$ is a perfect square.
2. $n^{\alpha-\beta+2\gamma}y = xz^2$.
3. $\dfrac{6n^{\beta-\alpha}y}{x}$ is a nasty number.
4. $\dfrac{n^{\alpha-\beta}y}{x}$ is a perfect square.

Conclusion:

In this paper, we have presented a method to obtain infinitely many integer solutions to the generalized Fermat equations involving jarasandha numbers. To conclude that, one may search for other choices of generalized Fermat equations with some other numbers.

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